Phys 375 HW 6
Fall 2009
Due 16 / 17 November, 2009

1. A single slit in an opaque screen 0.10 mm wide is illuminated (in air) by plane waves from a krypton ion laser (λ_0 = 461.9 nm). If the observing screen is 1.0 m away, determine whether or not the resulting diffraction pattern will be of the far-field variety and then compute the angular width of the central maximum.

Solution:

Far field condition is given by $R > \frac{b^2}{\lambda}$, *where R is the distance from slit to observation point and b is the slit width. R=1.0 m, and plugging in the numbers we find:* $\frac{b^2}{\lambda} = 0.02m$. *YES, it is far field. From the diffraction formula we know the first zero of intensity is given by* $\sin\theta = \lambda / b$. *Implying:*

$$\theta = \sin^{-1}(\lambda / b) = 0.26^\circ$$

And the angular width is given by: $2\theta = 0.52^\circ$.

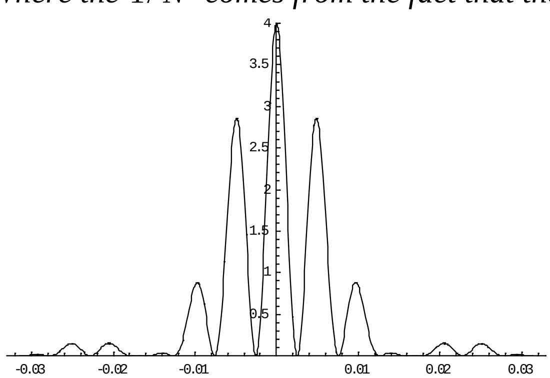
2. What is the relative irradiance of the subsidiary maxima in a three-slit Fraunhofer diffraction pattern? Draw a graph of the irradiance distribution, when the split spacing *a* = 2*b*, where *b* is the slit width, for 2 and then 3 slits.

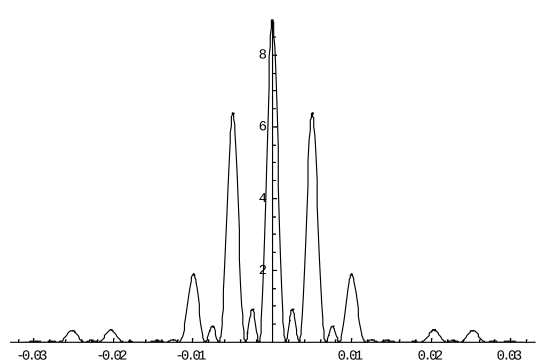
Solution:

The multi-slit diffraction formula is $I = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$. *The subsidiary maxima occur for* $N\alpha = p\pi / 2$, *with p an odd integer and* $\sin\alpha \neq 0$. *Thus the first subsidiary maximum occurs of p=3 and we have N=3, thus* $\alpha = \pi / 2$. *Using the fact that* $\alpha / \beta = a / b$ *and from the diffraction formula we find:*

$$\frac{I(\theta)}{I_0} = \frac{1}{N^2}\left(\frac{\sin\beta}{\beta}\right)^2\Bigg|_{\alpha=\pi/2} \approx 1/9$$

Where the $1/N^2$ *comes from the fact that the maximum I is given by* $N^2 I_0$.





Above are plots of the diffraction pattern for a (left) 2-slit (b = 0.04 mm, a = .125 mm) and (right) 3-slit (b = 0.04 mm, a = .125 mm) illuminated with He-Ne laser light at 632.8 nm.

4. Pedrotti[3], 3rd edition, problem 11-3. See Fig. 11-19 on page 290.

Solution:

See figure 11-19.

a) The positions of the minima are given by: $m\lambda = b\sin\theta_m \approx b\theta_m = by_m / L$. *Where L=2m and* y_m *is the location of the m-th zero. Thus* $\Delta y = y_3 - y_{-3} = 6\lambda L / b$. *Upon plugging in the numbers we find* $b = 0.13mm$.

b) Again $L > \dfrac{b^2}{\lambda}$ *is the far field condition and* $\dfrac{L}{b^2/\lambda} = 139$. *YES this is far field.*

5. Pedrotti[3], 3rd edition, problem 11-5

Solution:

The full angular breadth of the central maximum is given by $\varphi = 2\theta$, *where* θ *is the position of the first zero. Thus,*

$$b = \frac{\lambda}{\sin(\varphi/2)}$$

For $\varphi = 30^\circ, 45^\circ, 90^\circ, 180^\circ$ *we have* $b = 2.125mm, 1.437\mu m, 0.778\mu m, 0.55\mu m$ *respectively.*

3. Pedrotti[3], 3rd edition, problem 11-11

Solution:

The Airy disc formed by a circular diffraction aperture of diameter D *has angular radius:* $\Delta\theta_{1/2} = \dfrac{1.22\lambda}{D}$. *Thus the radius R of the Airy disc formed is given by:*

$$R = L\tan\Delta\theta_{1/2} \approx L\Delta\theta_{1/2} = 4.86\times10^6\, m$$

The irradiance is then given by:

$$I = \Phi / A = \frac{2000W}{\pi\left(4.86\times10^6\right)^2} = 2.7\times10^{-11} W/m^2$$

6. Pedrotti[3], 3rd edition, problem 11-14

Solution:

a) The maximum and minimum distances are for a line separation of s =1mm are:

Minimum: $\Delta\theta = \dfrac{s}{L} = \dfrac{1.22\lambda}{D}$, *implies* $L = 3.0m$ *and Maximum:* $L = 10.4m$.

b) The pupil diameter will be $D = \dfrac{1.22\lambda L}{s} = 6.71\times10^{-4}\,L$